

Indian Statistical Institute, Bangalore

B.Math (Hons.)III Year, First Semester

Mid-Sem Examination

Differential Geometry

Time: 3 hours

Date:27 Sept 2010

Instructor: B.Bagchi

Maximum Marks 100

1. A cycloid is the plane curve traced out by a point on the circumference of a circle as it rolls without slipping along a straight line. If the circle is of radius one and the straight line is the x - axis, find a formula for the cycloid as a function of the time parameter. Hence find the length of the cycloid corresponding to one complete revolution of the circle. [20]
2. Define a regular curve and show that any regular curve has a unit speed reparametrization. Find the unit speed parametrization of the curve $\gamma(t) = (t, \cosh t)$, $-\infty < t < \infty$. [20]
3. Define the signed and unsigned curvature of a regular plane curve. Show the the unsigned curvature of the regular plane curve γ is given by the formula $k(t) = \frac{\|\gamma'(t) \times \gamma''(t)\|}{\|\gamma'(t)\|^3}$. [20]
4. If $F : [0, \pi] \rightarrow \mathbb{R}$ is a smooth function with $F(0) = F(\pi) = 0$ then show that $\int_0^\pi (F'(t))^2 dt \geq \int_0^\pi (F(t))^2 dt$. Find all cases of equality. [20]
5. Use Archimedes' Theorem to find the area of a lune in the unit sphere. Hence find a formula for the area of a spherical triangle in the unit sphere in terms of its angles. [20]

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